

**Self-assessment: 14 Further differentiation**

1. Differentiate the following expressions with respect to  $x$ :

(a)  $(2x + 1)^5$

(b)  $\cos^3(2x)$

*(accessible to students on the path to grade 3 or 4) [4 marks]*

2. Do not use a calculator to answer this question.

Find the  $x$ -coordinate of the stationary point on the graph of  $y = \frac{e^{-2x}}{x^2}$ .

*(accessible to students on the path to grade 5 or 6) [5 marks]*

3. A closed cylinder has radius  $r$  cm and height  $h$  cm. Its volume is  $208 \text{ cm}^3$ .

(a) Find an expression for the surface area of the cylinder in terms of  $h$  only.

(b) Find the value of  $h$  which minimises the surface area of the cylinder.

*(accessible to students on the path to grade 5 or 6) [7 marks]*

4. Given that  $f(x) = x \sin(ax)$  with  $a > 0$ ,

(a) Find  $f'(x)$  and  $f''(x)$ .

*(accessible to students on the path to grade 5 or 6)*

(b) (i) Show that the stationary points of  $f(x)$  satisfy the equation  $\tan(ax) = -ax$ .

(ii) Use a graph to show that the above equation has only one solution for

$$x \in \left(-\frac{\pi}{2a}, \frac{\pi}{2a}\right).$$

(iii) Hence find the coordinates of the stationary point on the graph of  $y = f(x)$  and determine its nature.

*(accessible to students on the path to grade 7)*

(c) Find the value of  $a$  for which  $f(x)$  satisfies the equation  $f''(x) + 4f(x) = 2a \cos(ax)$ .

*(accessible to students on the path to grade 5 or 6) [14 marks]*